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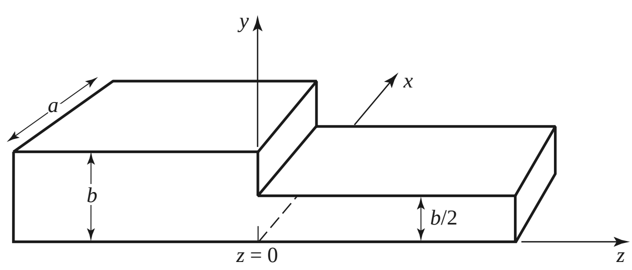
PROBLEMS

4.1 Consider the reflection of a TE_{10} mode, incident from $z < 0$, at a step change in the height of a rectangular waveguide, as shown below. Show that if the method of Example 4.2 is used, the result $\Gamma = 0$ is obtained. Do you think this is the correct solution? Why? (This problem shows that the one-mode impedance viewpoint does not always provide a correct analysis.)



4.2 Consider a series *RLC* circuit with a current I. Calculate the power lost and the stored electric and magnetic energies, and show that the input impedance can be expressed as in (4.17).

4.3 Show that the input impedance Z of a parallel *RLC* circuit satisfies the condition that $Z(-\omega) = Z^*(\omega)$.

4.4 A two-port network is driven at both ports such that the port voltages and currents have the following values ($Z_0 = 50\ \Omega$):

$$V_1 = 10\angle 90^\circ, \quad I_1 = 0.2\angle 90^\circ,$$
$$V_2 = 8\angle 0^\circ, \quad I_2 = 0.16\angle -90^\circ.$$

Determine the input impedance seen at each port, and find the incident and reflected voltages at each port.

4.5 Show that the admittance matrix of a lossless N-port network has purely imaginary elements.

4.6 Does a nonreciprocal lossless network always have a purely imaginary impedance matrix?

4.7 Derive the $[Z]$ and $[Y]$ matrices for the two-port networks shown in the figure below.

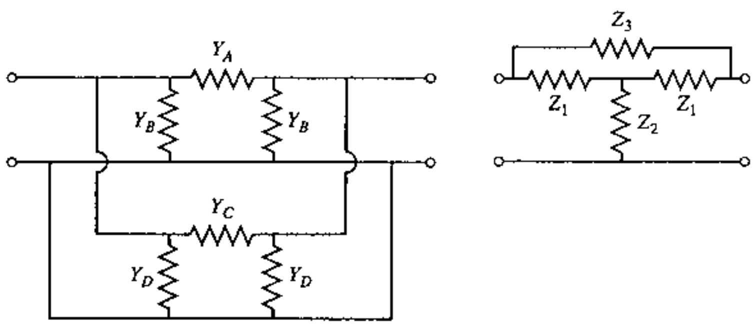
(a) (b)

4.8 Consider a two-port network, and let $Z_{SC}^{(1)}$, $Z_{SC}^{(2)}$, $Z_{OC}^{(1)}$, and $Z_{OC}^{(2)}$ be the input impedance seen when port 2 is short-circuited, when port 1 is short-circuited, when port 2 is open-circuited, and when port 1 is open-circuited, respectively. Show that the impedance matrix elements are given by

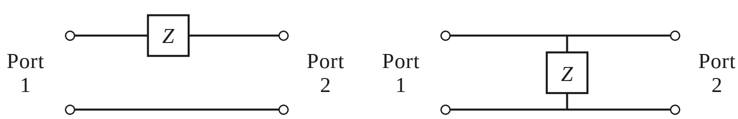
$$Z_{11} = Z_{OC}^{(1)}, \quad Z_{22} = Z_{OC}^{(2)}, \quad Z_{12}^2 = Z_{21}^2 = \left(Z_{OC}^{(1)} - Z_{SC}^{(1)}\right) Z_{OC}^{(2)}.$$

4.9 Find the impedance parameters of a section of transmission line with length ℓ, characteristic impedance Z_0, and propagation constant β.

4.10 Show that the admittance matrix of the two parallel-connected two-port π networks shown below can be found by adding the admittance matrices of the individual two-ports. Apply this result to find the admittance matrix of the bridged-T circuit shown. What is the corresponding result for the impedance matrix of two series-connected T-networks?



4.11 Find the scattering parameters for the series and shunt loads shown below. Show that $S_{12} = 1 - S_{11}$ for the series case, and that $S_{12} = 1 + S_{11}$ for the shunt case. Assume a characteristic impedance Z_0.



4.12 Consider two two-port networks with individual scattering matrices $[S^A]$ and $[S^B]$. Show that the overall S_{21} parameter of the cascade of these networks is given by

$$S_{21} = \frac{S_{21}^A S_{21}^B}{1 - S_{22}^A S_{11}^B}.$$

4.13 Consider a lossless two-port network. (a) If the network is reciprocal, show that $|S_{21}|^2 = 1 - |S_{11}|^2$. (b) If the network is nonreciprocal, show that it is impossible to have unidirectional transmission, where $S_{12} = 0$ and $S_{21} \neq 0$.

4.14 A four-port network has the scattering matrix shown as follows. (a) Is this network lossless? (b) Is this network reciprocal? (c) What is the return loss at port 1 when all other ports are terminated with matched loads? (d) What is the insertion loss and phase delay between ports 2 and 4 when all other ports are terminated with matched loads? (e) What is the reflection coefficient seen at port 1 if a short circuit is placed at the terminal plane of port 3 and all other ports are terminated with matched loads?

$$[S] = \begin{bmatrix} 0.178\angle 90^\circ & 0.6\angle 45^\circ & 0.4\angle 45^\circ & 0 \\ 0.6\angle 45^\circ & 0 & 0 & 0.3\angle -45^\circ \\ 0.4\angle 45^\circ & 0 & 0 & 0.5\angle -45^\circ \\ 0 & 0.3\angle -45^\circ & 0.5\angle -45^\circ & 0 \end{bmatrix}.$$

4.15 Show that it is impossible to construct a three-port network that is lossless, reciprocal, and matched at all ports. Is it possible to construct a nonreciprocal three-port network that is lossless and matched at all ports?

4.16 Prove the following *decoupling theorem*: For any lossless reciprocal three-port network, one port (say port 3) can be terminated in a reactance so that the other two ports (say ports 1 and 2) are decoupled (no power flow from port 1 to port 2, or from port 2 to port 1).

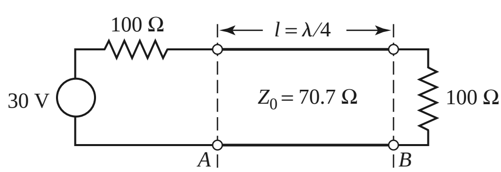
4.17 A certain three-port network is lossless and reciprocal, and has $S_{13} = S_{23}$ and $S_{11} = S_{22}$. Show that if port 2 is terminated with a matched load, then port 1 can be matched by placing an appropriate reactance at port 3.

4.18 A four-port network has the scattering matrix shown as follows. If ports 3 and 4 are connected with a lossless matched transmission line with an electrical length of 45°, find the resulting insertion loss and phase delay between ports 1 and 2.

$$[S] = \begin{bmatrix} 0.2\angle 50^\circ & 0 & 0 & 0.4\angle -45^\circ \\ 0 & 0.6\angle 45^\circ & 0.7\angle -45^\circ & 0 \\ 0 & 0.7\angle -45^\circ & 0.6\angle 45^\circ & 0 \\ 0.4\angle -45^\circ & 0 & 0 & 0.5\angle 45^\circ \end{bmatrix}.$$

4.19 When normalized to a single characteristic impedance Z_0, a certain two-port network has scattering parameters S_{ij}. Find the generalized scattering parameters, S_{ij}^p, in terms of the real reference impedances, R_{01} and R_{02}, at ports 1 and 2, respectively.

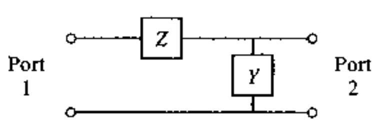
4.20 At reference plane A, for the circuit shown below, choose an appropriate reference impedance, find the power wave amplitudes, and compute the power delivered to the load. Repeat this procedure for reference plane B. Assume the transmission line is lossless.



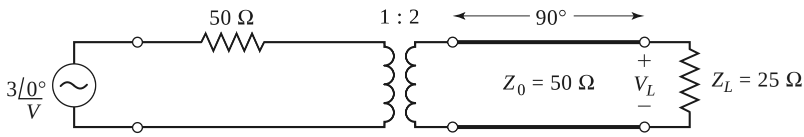
4.21 The *ABCD* parameters of the first entry in Table 4.1 were derived in Example 4.6. Verify the *ABCD* parameters for the second, third, and fourth entries.

4.22 Derive expressions that give the impedance parameters in terms of the *ABCD* parameters.

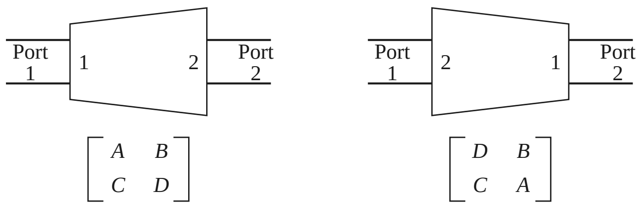
4.23 Find the *ABCD* matrix for the circuit shown below by direct calculation using the definition of the *ABCD* matrix, and compare with the *ABCD* matrix of the appropriate cascade of canonical circuits from Table 4.1.



4.24 Use *ABCD* matrices to find the voltage V_L across the load resistor in the circuit shown below.

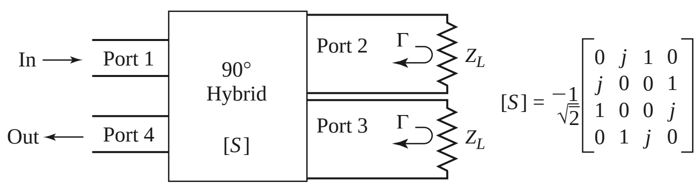


4.25 A reciprocal two-port network with its *ABCD* matrix is shown below at left. Prove that the network with ports 1 and 2 in reversed positions has the *ABCD* matrix shown below at right. Choose a simple asymmetrical network to demonstrate this result.

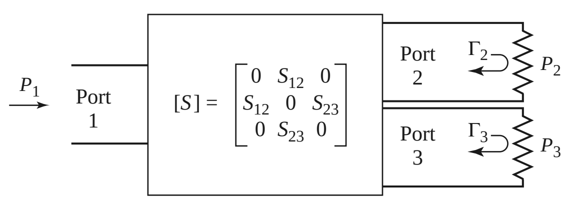


4.26 Derive the expressions for S parameters in terms of the *ABCD* parameters, as given in Table 4.2.

4.27 As shown in the figure below, a variable attenuator can be implemented using a four-port 90° hybrid coupler by terminating ports 2 and 3 with equal but adjustable loads. (a) Using the given scattering matrix for the coupler, show that the transmission coefficient between the input (port 1) and the output (port 4) is given as $T = j\Gamma$, where Γ is the reflection coefficient of the mismatch at ports 2 and 3. Also show that the input port is matched for all values of Γ. (b) Plot the attenuation, in dB, from the input to the output as a function of Z_L/Z_0, for $0 \leq Z_L/Z_0 \leq 10$ (let Z_L be real).



4.28 Use signal flow graphs to find the power ratios P_2/P_1 and P_3/P_1 for the mismatched three-port network shown in the accompanying figure.



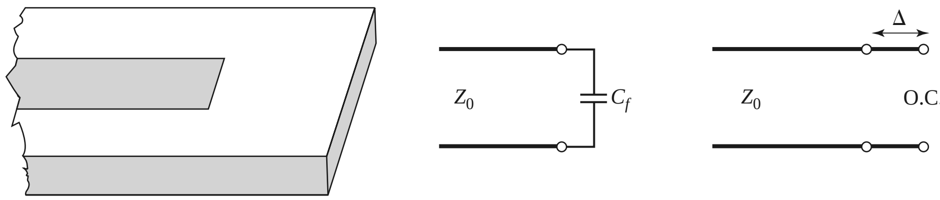
4.29 The *ABCD* parameters are useful for treating cascades of two-port networks in terms of the total port voltages and currents, but it is also possible to use incident and reflected voltages to treat cascades. One way of doing this is with the *transfer*, or *T*-, *parameters*, defined as follows:

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} b_2 \\ a_2 \end{bmatrix},$$

where a_1, b_1 and a_2, b_2 are the incident and reflected voltages at ports 1 and 2, respectively. Derive the T-parameters in terms of the scattering parameters of a two-port network. Show how the T-parameters can be used for a cascade of two two-port networks.

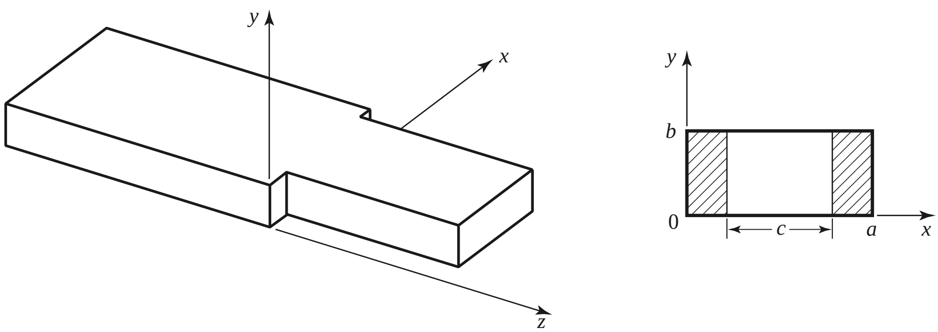
4.30 The end of an open-circuited microstrip line has fringing fields that can be modeled as a shunt capacitor, C_f, at the end of the line, as shown below. This capacitance can be replaced with an additional length, Δ, of microstrip line. Derive an expression for the length extension in terms of the fringing capacitance. Evaluate the length extension for a 50 Ω open-circuited microstrip line on a substrate with $d = 0.158$ cm and $\epsilon_r = 2.2$ ($w = 0.487$ cm, $\epsilon_e = 1.894$), if the fringing capacitance is known to be $C_f = 0.075$ pF. Compare your result with the approximation given by Hammerstad and Bekkadal:

$$\Delta = 0.412d \left(\frac{\epsilon_e + 0.3}{\epsilon_e - 0.258}\right)\left(\frac{w + 0.262d}{w + 0.813d}\right).$$



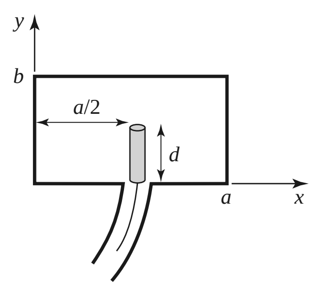
4.31 For the H-plane step analysis of Section 4.6, compute the complex power flow in the reflected modes in guide 1, and show that the reactive power is inductive.

4.32 Derive the modal analysis equations for the symmetric H-plane step shown below. (HINT: Because of symmetry, only the TE_{n0} modes for n odd will be excited.)

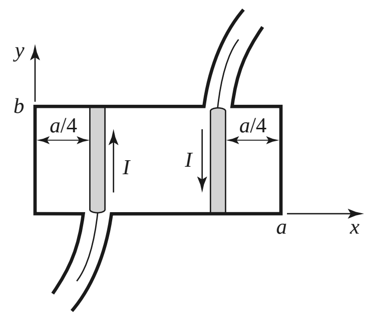


4.33 Find the transverse $\bar{E}$ and $\bar{H}$ fields excited by the current of (4.116) by postulating traveling TM_{mn} modes on either side of the source at $z = 0$ and applying the appropriate boundary conditions.

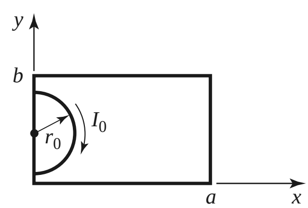
4.34 An infinitely long rectangular waveguide is fed with a probe of length d as shown below. The current on this probe can be approximated as $I(y) = I_0 \sin k(d - y)/\sin kd$. If the TE_{10} mode is the only propagating mode in the waveguide, compute the input resistance seen at the probe terminals.



4.35 Consider the infinitely long waveguide fed with two probes driven 180° out of phase, as shown below. What are the resulting excitation coefficients for the TE_{10} and TE_{20} modes? What other modes can be excited by this feeding arrangement?



4.36 Consider a small current loop on the sidewall of a rectangular waveguide, as shown below. Find the TE_{10} fields excited by this loop if the loop is of radius r_0.



4.37 A rectangular waveguide is shorted at $z = 0$ and has an electric current sheet, J_{sy}, located at $z = d$, where

$$J_{sy} = \frac{2\pi A}{a} \sin \frac{\pi x}{a}$$

(see the accompanying figure). Find expressions for the fields generated by this current by assuming standing wave fields for $0 < z < d$, and traveling wave fields for $z > d$, and applying boundary conditions at $z = 0$ and $z = d$. Now solve the problem using image theory, by placing a current sheet $-J_{sy}$ at $z = -d$, and removing the shorting wall at $z = 0$. Use the results of Section 4.7 and superposition to find the fields radiated by these two currents, which should be the same as the first results for $z > 0$.